Why use Binary?

- It is a two state system (on/off) which makes it simple to operate

- There are only four rules for addition in binary

  [0+0=0 ; 0+1=1 ; 1+0=1; 1+1=10]

Number Systems - Decimal

- The decimal system is a base-10 system. There are 10 distinct digits (0 to 9) to represent any quantity.
- Decimal numbers are made of decimal digits: (0,1,2,3,4,5,6,7,8,9)
- For an n-digit number, the value that each digit represents depends on its weight or position. The weights are based on powers of 10.

- a decimal number represent?
  - $8653 = 8x10^3 + 6x10^2 + 5x10^1 + 3x10^0$
- What about fractions?
  - $97654.35 = 9x10^4 + 7x10^3 + 6x10^2 + 5x10^1 + 4x10^0 + 3x10^{-1} + 5x10^{-2}$

4TH.	3RD.	2ND.	1ST.	POSITION
$10^3 = 1000$	$10^2 = 100$	$10^1 = 10$	$10^0 = 1$	WEIGHT

For example, 4916_{10} = 4*1000 + 9*100 + 1*10 +6*1

Number Systems - Binary

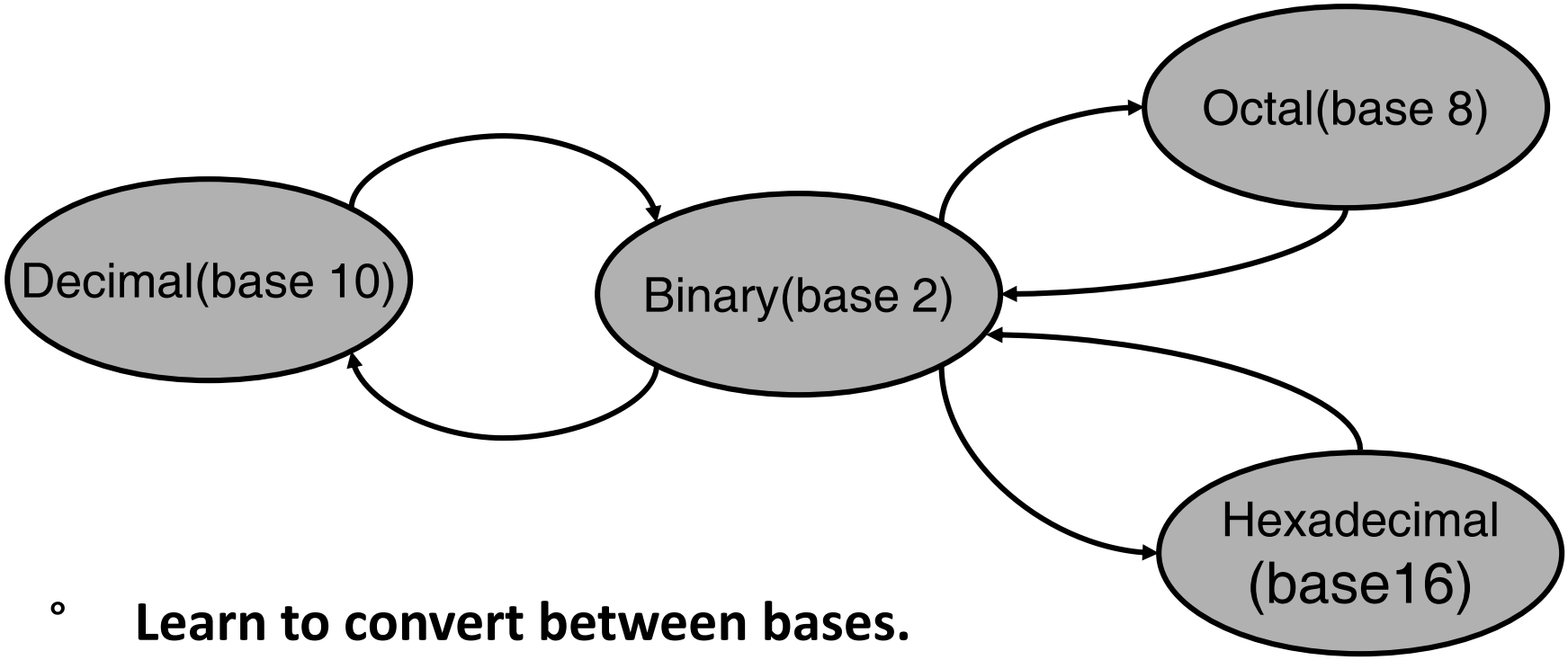
- The binary system is a base-2 system. There are 2 distinct digits (0 and 1) to represent any quantity.
- Binary numbers are made of binary digits (bits):
  - 0 and 1
- To express any number in base 2 we use powers much like our own decimal system.

- How many items does an binary number represent?
  - $(1011)_2 = 1x2^3 + 0x2^2 + 1x2^1 + 1x2^0 = (11)_{10}$
- What about fractions?
  - $(110.10)_2 = 1x2^2 + 1x2^1 + 0x2^0 + 1x2^{-1} + 0x2^{-2}$
- Groups of eight bits are called a *byte*
  - $(11001001)_2$
- Groups of four bits are called a *nibble.*
  - $(1101)_2$

8TH	7TH	6TH	5TH	4TH	3RD	2ND	1ST	POSITION
$2^7=128$	$2^6=64$	$2^5=32$	$2^4=16$	$2^3=8$	$2^2=4$	$2^1=2$	$2^0=1$	WEIGHT

For example: $11010010_2 = 1*128 + 1*64 + 0*32 + 1*16 + 0*8 + 0*4 + 1*2 + 0*1 = 210_{10}$

Conversion Between Number Bases



- **Learn to convert between bases.**

Number Systems - Binary to Decimal

- Converting binary to decimal

$2^7=128$	$2^6=64$	$2^5=32$	$2^4=16$	$2^3=8$	$2^2=4$	$2^1=2$	$2^0=1$		
1	**0**	**1**	**1**	**1**	**0**	**0**	**1**		

1x128+0x64+1x32+1x16+1x8+0x4+0x2+1

= 128+32+16+8+1

= 185

Convert an Integer *from* Decimal *to* Another Base

For each digit position:

1. **Divide decimal number by the base (e.g. 2)**
2. **The *remainder* is the lowest-order digit**
3. **Repeat first two steps until no *divisor* remains.**

Example for $(13)_{10}$:

	Integer Quotient		Remainder	Coefficient
13/2 =	6	+	½	$a_0 = 1$
6/2 =	3	+	0	$a_1 = 0$
3/2 =	1	+	½	$a_2 = 1$
1/2 =	0	+	½	$a_3 = 1$

Answer $(13)_{10} = (a_3\, a_2\, a_1\, a_0)_2 = (1101)_2$

Convert an Fraction *from* Decimal *to* Another Base

For each digit position:

1. **Multiply decimal number by the base (e.g. 2)**
2. **The *integer* is the highest-order digit**
3. **Repeat first two steps until fraction becomes zero.**

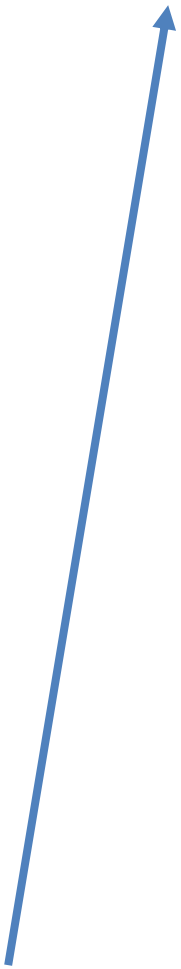
Example for $(0.625)_{10}$:

	Integer		Fraction	Coefficient
0.625 x 2 =	1	+	0.25	$a_{-1} = 1$
0.250 x 2 =	0	+	0.50	$a_{-2} = 0$
0.500 x 2 =	1	+	0	$a_{-3} = 1$

Answer $(0.625)_{10} = (0.a_{-1}\, a_{-2}\, a_{-3})_2 = (0.101)_2$

The Growth of Binary Numbers

n	2^n
0	$2^0=1$
1	$2^1=2$
2	$2^2=4$
3	$2^3=8$
4	$2^4=16$
5	$2^5=32$
6	$2^6=64$
7	$2^7=128$



n	2^n	
8	$2^8=256$	
9	$2^9=512$	
10	$2^{10}=1024$	
11	$2^{11}=2048$	
12	$2^{12}=4096$	
20	$2^{20}=1M$	Mega
30	$2^{30}=1G$	Giga
40	$2^{40}=1T$	Tera

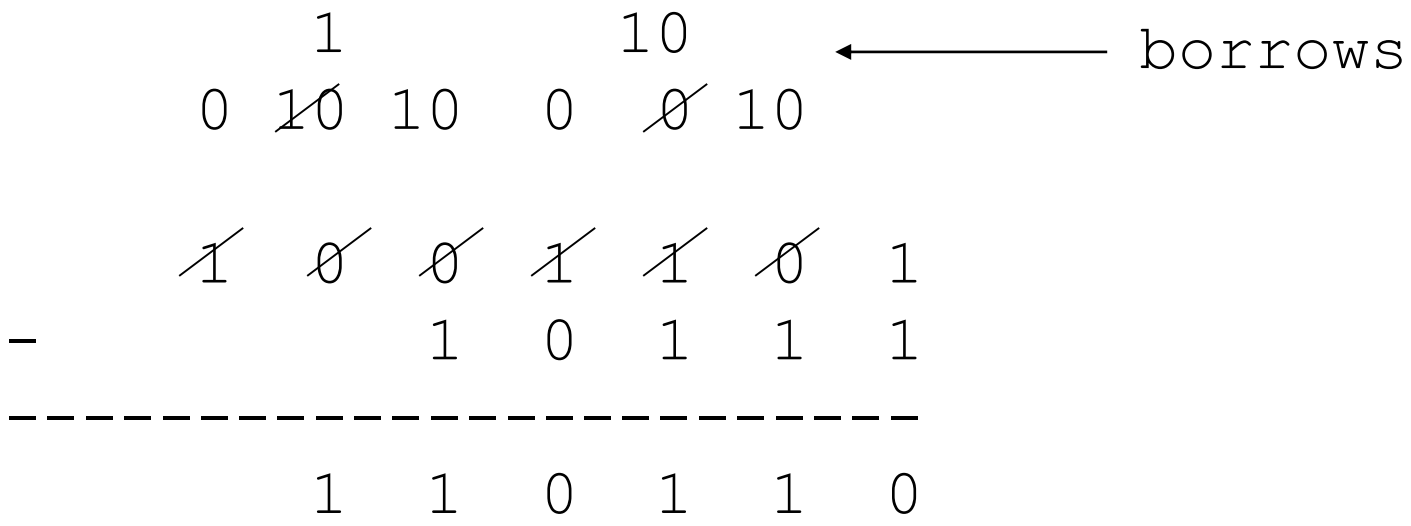
Binary Addition

- Binary addition is very simple.
- This is best shown in an example of adding two binary numbers…

```
   1  1  1  1  1  1 <------------ carries
      1  1  1  1  0  1
+     1  0  1  1  1
---------------------
   1  0  1  0  1  0  0
```

Binary Subtraction

- **We can also perform subtraction (with borrows in place of carries).**
- **Let's subtract $(10111)_2$ from $(1001101)_2$…**



Binary Multiplication

- Binary multiplication is much the same as decimal multiplication, except that the multiplication operations are much simpler…

```
                1   0   1   1   1
X                   1   0   1   0
-------------------------------
                0   0   0   0   0
            1   0   1   1   1
        0   0   0   0   0
    1   0   1   1   1
-----------------------------
    1   1   1   0   0   1   1   0
```

Convert an Integer *from* Decimal *to* Octal

For each digit position:

1. **Divide decimal number by the base (8)**
2. **The *remainder* is the lowest-order digit**
3. **Repeat first two steps until no *divisor* remains.**

Example for $(175)_{10}$:

	Integer Quotient		Remainder	Coefficient
175/8 =	21	+	7/8	$a_0 = 7$
21/8 =	2	+	5/8	$a_1 = 5$
2/8 =	0	+	2/8	$a_2 = 2$

Answer $(175)_{10} = (a_2\, a_1\, a_0)_2 = (257)_8$

Convert an Fraction *from* Decimal *to* Octal

For each digit position:

1. **Multiply decimal number by the base (e.g. 8)**
2. **The *integer* is the highest-order digit**
3. **Repeat first two steps until fraction becomes zero.**

Example for $(0.3125)_{10}$:

	Integer		Fraction	Coefficient
0.3125 x 8 =	2	+	5	$a_{-1} = 2$
0.5000 x 8 =	4	+	0	$a_{-2} = 4$

Answer $(0.3125)_{10} = (0.24)_8$

Summary

- Binary numbers are made of binary digits (bits)
- Binary and octal number systems
- Conversion between number systems
- Addition, subtraction, and multiplication in binary

