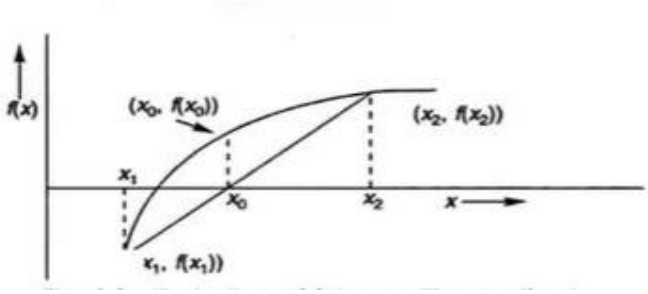
False Position Method

The false position method is a type of bracketing method, which means that it always generates a sequence of intervals that contains the root. It is guaranteed to converge to a root if the function is continuous and changes sign within the interval [a,b]. However, the method may converge very slowly or not at all if the function is highly non-linear or has multiple roots within the interval.

In bisection method, the interval between x_1 and x_2 is divided into two equal halves, irrespective of location of the root. It may be possible that the root is closer to one end than the other. In the figure below, the root is closer to x_1. When the two points, x_1 and x_2 are joined by the straight line, the point of intersection of this line with the x-axis (at x_0) gives an improved estimate of the root and is called the **false position** of the root. This point then replaces one of the initial guesses that has a function value of the same sign as $f(x_0)$. The process is repeated with the new values of x_1 and x_2. This method is also known as **Falsi** method and often known as **linear interpolation method** (because an approximate root is determined by linear interpolation).



Here, the equation of the line is given by:

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} * (x - x_1)$

$y - f(x_1) = \frac{f(x_2) - f(x_1)}{x_2 - x_1} * (x - x_1)$

From the figure, at $x = x_0, y = 0$

$0 - f(x_1) = \frac{f(x_2) - f(x_1)}{x_2 - x_1} * (x_0 - x_1)$

$\frac{-f(x_1) * (x_2 - x_1)}{f(x_2) - f(x_1)} = (x_0 - x_1)$

$$\therefore x_0 = x_1 - \frac{f(x_1) * (x_2 - x_1)}{f(x_2) - f(x_1)}$$

Further, the same formula can be extended to:

$$x_0 = \frac{x_1 * f(x_2) - x_2 * f(x_1)}{f(x_2) - f(x_1)}$$

Once we obtain x_0, to calculate the roots, we have the following conditions:

1. If $f(x_0) = 0$ then *root* = x_0.
2. If $f(x_1)*f(x_0) < 0$, *root* lies between x_1 and x_0.
3. If $f(x_0)*f(x_2) < 0$, *root* lies between x_0 and x_2.

Algorithm

Step 1: Start

Step 2: Initialize the values of x_1, x_2, e

Step 3: Compute $f_1 = f(x_1)$ *and* $f_2 = f(x_2)$

Step 4: If $f_1 * f_2 > 0$, then display **wrong initial guess as roots are not within x_1 and x_2** and goto (2)

Else

Continue

Step 5: Calculate the false point as : $x_0 = \frac{x_1 * f(x_2) - x_2 * f(x_1)}{f(x_2) - f(x_1)}$

Step 6: If $f(x_1)*f(x_0) < 0$ then $x_2 = x_0$

Else $x_1 = x_0$

Step 7: If $|f(x_0)| > e$ then goto 5 else goto 8

Step 8: Display x_0 as root

Step 9: Stop

Examples

1. Find a real root of equation $x^3 - 2x - 5 = 0$ using False Position Method correct upto 3 decimal places.

   Solution:
   Given:
   $x^3 - 2x - 5 = 0$

Let, *f(x)* = $x^3 - 2x - 5$

Let, the initial guess i.e. x_1 = 2 and x_2=3.
Since, False Position method is the bracketing method, we need to check if the initial guess brackets the root or not.

Now, we calculate $f(x_1)$ and $f(x_2)$.

$f(x_1)$ = $2^3 - 2 * 2 - 5$

= 8 – 4 – 5

= -1

$f(x_2)$ = $3^3 - 2 * 3 - 5$

= 27 – 6 – 5

= 16

Since, $f(x_1) * f(x_2) < 0$, root lies between x_1 = 2 and x_2=3.

Iteration	x_1	x_2	$f(x_1)$	$f(x_2)$	$x_0 = \frac{x_1 * f(x_2) - x_2 * f(x_1)}{f(x_2) - f(x_1)}$	$f(x_0)$	***Update***
1	2	3	-1	16	2.0588	-0.39	$x_1=x_0$
2	2.0588	3	-0.39	16	2.0813	-0.14	$x_1=x_0$
3	2.0813	3	-0.14	16	2.0897	-0.05	$x_1=x_0$
4	2.0897	3	-0.05	16	2.0928	-0.02	$x_1=x_0$
5	2.0928	3	-0.02	16	2.0939	-0.007	$x_1=x_0$
6	2.0939	3	-0.007	16	2.0943	-0.002	$x_1=x_0$
7	2.0943	3	-0.002	16	2.0945	-0.001	$x_1=x_0$

Hence, the root is **2.0945** which is correct upto 3 decimal places.

2. Find a root of an equation *f(x)* =$x^3 + x^2 - 3x - 3$ using False Point Method, correct up to 3 decimal places.
   Solution:
   Given:
   f(x) = $x^3 + x^2 - 3x - 3$

   Let, the initial guess i.e. x_1 = 1 and x_2=2.

Since, False Point method is the bracketing method, we need to check if the initial guess brackets the root or not.

Now, we calculate $f(x_1)$ and $f(x_2)$.

$f(x_1) = 1^3 + 1^2 - 3 * 1 - 3$

= 1+1-3-3

= -4

$f(x_2) = 2^3 + 2^2 - 3 * 2 - 3$

= 8+4-6-3

= 3

Since, $f(x_1) * f(x_2) < 0$, root lies between $x_1 = 1$ and $x_2 = 2$.

Iteration	x_1	x_2	$f(x_1)$	$f(x_2)$	$x_0 = \frac{x_1 * f(x_2) - x_2 * f(x_1)}{f(x_2) - f(x_1)}$	$f(x_0)$	**Update**
1	1	2	-4	3	1.5714	-1.36	$x_1 = x_0$
2	1.5714	2	-1.36	3	1.7054	-0.24	$x_1 = x_0$
3	1.7054	2	-0.24	3	1.7279	-0.03	$x_1 = x_0$
4	1.7279	2	-0.03	3	1.7314	-0.006	$x_1 = x_0$
5	1.7314	2	-0.006	3	1.7320	-0.001	$x_1 = x_0$
6	1.7320	2	-0.001	3	1.7320	-0.0001	$x_1 = x_0$

Hence, the root is **1.7320** which is correct upto 3 decimal places.

Practice Questions:

1. Find the root of $e^{-x} - x = 0$ correct upto 3 decimal places using False Position method.
2. Find the root of $sinx = \frac{1}{x}$ correct upto 2 decimal places using False Position method.
3. Find the root of $x - \cos(x) = 0$ correct upto 4 decimal places using False Position method.
4. Find the root of $3x + \sin(x) - e^x = 0$ correct upto 4 decimal places using False Position method.
5. Find the root of $\ln(x) - x + 3 = 0$ correct upto 4 decimal places using False Position method.